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New Controllers Efficient Model-Based Design Method

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Abstract
This paper proposes a new simple and efficient model-based time domain P, PI ,PD , and PID controllers design methods for achieving an important design compromise; acceptable stability, and medium fastness of response, the proposed method is based on selecting controllers' gains based on plant's parameters, a simple expressions are proposed for calculating and soft tuning controller's gain, the proposed controllers design methods were tested for first, second and first order system with time delay, and using MATLAB/simulink software.
Keywords: Controller, controller design.

1. Introduction

The term control system design refers to the process of selecting feedback gains (poles and zeros) that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant (Ahmad A. Mahfouz, et al 2013). An important compromise for control system design is to result in acceptable stability, and medium fastness of response, one definition of acceptable stability is when the undershoot that follows the first overshoot of the response is small, or barely observable. Beside world wide known and applied controllers design method including Ziegler–Nichols, Chiein-Hrones-Reswick (CHR), Wang–Juang–Chan, Cohen-Coon, many controllers design methods have been proposed in different papers and texts including (Astrom K,J et al 1994)(Ashish Tewari, 2002)(Katsuhiko Ogata, 2010)(Norman S. Nise, 2011)(Gene F. Franklin, et al 2002)(Dale E. Seborg, et al, 2004)(Dingyu Xue et al, 2007)(Chen C.L et al, 1989)(R. Matousek, *2012*)(K. J. Astrom et al, 2001)(Susmita Das et al, 2012) (L. Ntogramatzidis, 2010)(M.Saranya et al, 2012), each method has its advantages, and limitations. (R. Matousek, *2012*) present multi-criterion optimization of PID controller by means of soft computing optimization method HC12. (K. J. Astrom et al, 2001) introduce an improved PID tuning approach using traditional Ziegler-Nichols tuning method with the help of simulation aspects and new built in function. (L. Ntogramatzidis et al, 2010) A unified approach has been presented that enable the parameters of PID, PI and PD controllers (with corresponding approximations of the derivative action when needed) to be computed in finite terms given appropriate specifications expressed in terms of steady-state performance, phase/gain margins and gain crossover frequency. (M.Saranya et al, 2012) proposed an Internal Model Control (IMC) tuned PID controller method for the DC motor for robust operation. (Fernando G. Martons, 2005) proposed a procedure for tuning PID controllers with simulink and MATLAB. (Saeed Tavakoli, 2003) presented Using dimensional analysis and numerical optimization techniques, an optimal method for tuning PID controllers for first order plus time delay systems.

This paper proposes P, PI, PD, and PID controller design method based on selecting controller gainsbased on plant's parameters that meet an important design compromise; acceptable stability, and medium fastness of response. By relating controller's gains and plant parameters, particularly, time constant, damping ratio and undamped natural frequency, expressions for selecting values of controllers' gains are to be derived for FOPTD, first and second order systems, as well as, systems that can be approximated as first and second order systems, to achieve more smooth response with minimum overshoot, minimum settling time, and minimum steady state error, a soft tuning parameters with recommended ranges are to be introduced.

1.1 Controllers Modeling

The controller that will be considered are Proportional, proportional derivative, proportional integral and proportional integral derivative controllers

Proportional Control: The control action of P-controller is proportional to the error, The relation between the *output* control signal of controller, *u(t)* and the actuating error signal *e(t)* is given by Eq.(1), taking Laplace-transform and manipulating Eq.(1), for transfer function gives:

$u(t) = K_p e(t)$ 膝臉 $(s) = E(s) K_p$ (1)

$G_p(s) = U(s)/E(s) = K_p$ (2)

Proportional-Derivative, PD-controller: The output control signal of PD-Controller controller *u(t),* is equal to the *sum* of two signals and given by Eq.(3), taking Laplace transform and solving for transfer function, gives

Eq.(4) :

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} \Leftrightarrow U(s) = K_P E(s) + K_D sE(s) \tag{3}$$

$$G_{PD}(s) = K_P + K_D s = K_D (s + \frac{K_P}{K_D}) = K_D (s + Z_{PD}) \tag{4}$$

Where: $Z_{PD} = K_P/K_D$, is the PD-controller zero**,**

Proportional-Integral, PI-controller: The output control action signal *u(t),* of PI- controller is proportional to the error and the integral of error, where the integral of the error, as well as, the error itself are used for control**,** and given by Eq.(5), taking Laplace transform, and solving for transfer function gives Eq.(6):

$$u(t) = K_P e(t) + K_I \int de(t)dt \Leftrightarrow U(s) = K_P E(s) + K_I E(s)\frac{1}{s} = E(s)\left(K_P + \frac{K_I}{s}\right) \tag{5}$$

$$G_{PI}(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s} = \frac{K_P (s + \frac{K_I}{K_P})}{s} = \frac{K_P (s + Z_{PI})}{s} \tag{6}$$

Where: $Z_{PI} = K_I / K_P$, is the PI-controller zero. Equation (6) can be rewritten, in terms of integral time constant T_I, to have the following given by (7), and implemented as shown in Figure 15(b) :

$$G_{PI}(s) = K_P + \frac{K_I}{s} = K_P (1 + \frac{K_I}{K_P s}) = K_P (1 + \frac{1}{T_I s}) \Leftrightarrow u(t) = K_P (e(t) + \frac{1}{Ti}\int e(t)dt)$$

$$u(t) = K_P (e + \frac{e}{Ti}) \tag{7}$$

This means if constant error exists, the controller action will keep increasing, until the error is zero, where: $T_I=K_P/K_I$, is the time constants of the integral actions*,* or *integral time* .

Proportional-Integral- Derivative PID-controller: Combining all three controllers, results in the PID controller, the output of PID controller is equal to the sum of three signals and given by Eq.(32), taking Laplace transform, and solving for transfer function , gives Eq.(8)

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int e(t)dt \Leftrightarrow U(s) = K_P E(s) + K_D E(s)s + K_I E(s)\frac{1}{s}$$

$$U(s) = E(s)\left[K_P + \frac{K_I}{s} + K_D s\right] \Rightarrow G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s \tag{8}$$

This equation can be manipulated to result in the form given by Eq.(9)

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} \tag{9}$$

Equation (9) is second order system, with two zeros and one pole at origin, and can be expressed to have the form given by Eq.(10), which indicates that PID transfer function is the product of transfer functions PI and PD , Implementing these two controllers jointly and independently will take care of both controller design requirements

$$G_{PID} = \frac{K_D (s + Z_{PI})(s + Z_{PD})}{s} = K_D (s + Z_{PI})\frac{(s + Z_{PD})}{s} = G_{PD}(s)G_{PI}(s) \tag{10}$$

The transfer function of PID controller, also ,can also be expressed to have the form given by Eq.(11):

$$G_{PID} = \frac{K_D (s + Z_{PI})(s + Z_{PD})}{s} = \frac{K_D s^2 + (Z_{PI} + Z_{PD})K_D s + (Z_{PI} Z_{PD} K_D)}{s} \tag{11}$$

Rearranging Eq.(11), we have:

$$G_{PID} = \frac{K_D s^2}{s} + \frac{(Z_{PI} + Z_{PD})K_D s}{s} + \frac{(Z_{PI} Z_{PD} K_D)}{s} = (Z_{PI} + Z_{PD})K_D + \frac{(Z_{PI} Z_{PD} K_D)}{s} + K_D s$$

Substituting the following values, $K_1 = (Z_{PI} + Z_{PD})K_D, K_2 = Z_{PI} Z_{PD} K_D), K_3 = K_D$, gives:

$$G_{PID} = K_1 + \frac{K_2}{s} + K_3 s \tag{12}$$

Since PID transfer function is a second order system, it can be expressed in terms of damping ratio and undamped natural frequency to have the form given by Eq.(13)::

$$G_{PID}(s)=\frac{K_D\left[s^2+\frac{K_P}{K_D}s+\frac{K_I}{K_D}\right]}{s}=\frac{K_D\left[s^2+2\xi\omega_n s+\omega_n^2\right]}{s} \qquad (13)$$

Where: $\omega_n^2=\frac{K_I}{K_D}$ and $2\xi\omega_n=\frac{K_P}{K_D}$

The transfer function of PID control given by Eq.(8) can, also, be expressed in terms of derivative time and integral time to have the form given by Eq.(14):

$$G_{PID}=K_P\left(1+\frac{1}{T_I s}+T_D s\right)=K_P\frac{T_I T_D s^2+T_I s+1}{T_I s} \qquad (14)$$

Where: $T_I=K_P/K_I$: *integral time,* $T_D=K_D/K_P$: *derivative time,*

$K_I=K_P/T_I$, $K_D=K_P T_D$

Since in Eq. (14) the numerator has a higher degree than the denominator, the transfer function is not causal and can not be realized, therefore this PID controller is modified through the addition of a lag to the derivative term, to have the following form:

$$G_{PID}=K_P\left(1+\frac{1}{T_I s}+\frac{T_D s}{1+\frac{T_D s}{N}}\right)$$, $_D$ /N - time constant of the added lag

Where: N: determines the gain K_{HF} of the PID controller in the high frequency range, the gain K_{HF} must be limited because measurement noise signal often contains high frequency components and its amplification should be limited. Usually, the divisor N is chosen in the range 2 to 20. If no D-controller, then we have PI controller, given by Eq. (15), it is clear that, PI and PD controllers are special cases of the PID controller.

$$G_{PI}=K_P\left(1+\frac{1}{T_I s}\right)=K_P\left(\frac{T_I s+1}{T_I s}\right) \qquad (15)$$

The addition of the proportional and derivative components effectively predicts the error value at T_D seconds (or samples) in the future, assuming that the loop control remains unchanged. The integral component adjusts the error value to compensate for the sum of all past errors, with the intention of completely eliminating them in T_I seconds (or samples). The resulting compensated single error value is scaled by the single gain K_P.

1.2 Dominant features

Most complex systems have dominant features that typically can be approximated by either a first or second order system response. Control system's response is largely dictated by those poles that are the closest to the imaginary axis, i.e. the poles that have the smallest real part magnitudes, such poles are called the dominant poles, many times, it is possible to identify a single pole, or a pair of poles, as the dominant poles. In such cases, a fair idea of the control system's performance can be obtained from the damping ratio and undamped natural frequency of the dominant poles (Farhan A. Salem, *2013*). For complex system, the controller gains are to be selected based on plant's parameters (time constant, the damping ratio and undamped natural frequency) of the dominant poles.

2. Controllers design for first order systems

First order systems and systems that can be approximated as first order systems, are characterized, *mainly*, by time constant *T*. Time constant is a characteristic time that is used as a measure of speed of response to a step input and governs the approach to a steady-state value after a long time. The general form of first order system's transfer function in terms of time constant T, is given by Eq.(16).

$$G(s)=\frac{1}{Ts+1} \qquad (16)$$

2.1 P-controller design for first order systems

To design P-controller, with minim settling time, the proportional gain K_P, is set equals to time constant, but since different *step* input values can be applied, and correspondingly different steady state value will results, then by multiplying plant's time constant *T*, by desired output value (reference input) the value of K_P can be selected to achieve suitable response. To soften the response in terms of reducing overshoot, settling time and steady state error, a tuning parameters *α* can be introduced, based on all this expression given by Eq.(17) can be proposed for designing P-controller for first order systems:

$$K_P=\alpha RT \qquad (17)$$

Where: T : Plant's time constant. R : desired output value, α: a tuning parameter that takes the value from 1 to 100. To achieve overall system fast response, with minimum overshoot and oscillation, parameter α, is softly increased.

For systems with small DC gain *and/or* small time constant, to minimize time of design and selection process of proportional gain K_P, the expression given by Eq.(18) can be applied directly, where each increase of parameter α will increase proportional gain by 100 factor

To test and verify the proposed P-controller design expression for selecting proportional gain K_P for achieving desired output of 10 with minimum possible overshoot, steady state error and settling time, four different first order systems with different time constants, with transfer functions given by Eq.(19) and unity feedback control systems simulink model shown in Figure 1 are to be used. The result of designing P-controller and tuned values of parameter α, for *first three* systems, are shown in Table 1, the response curves are shown in Figure 2, these response curves show that, acceptable stability, and medium fastness of response with minimum steady state error can be achieved by applying proposed expression, and by selecting and increasing the value of only one parameter α.

To test expression given by Eq.(18), applying it to system 3, and comparing with original expression, (see figure 2(c)(d)), the comparison shows that applying Eq.(18) to systems with small DC gain and/or small time constant, simplifies and accelerate design process while achieving optimal response.

$$K_P = 10\alpha RT \qquad (18)$$

$$G_{sys1}(s) = \frac{10}{10s+1}, \quad G_{sys2}(s) = \frac{1}{s+2} \quad G_{sys3}(s) = \frac{0.005}{10s+1} \quad G_{sys4}(s) = \frac{0.1}{2s+100} \qquad (19)$$

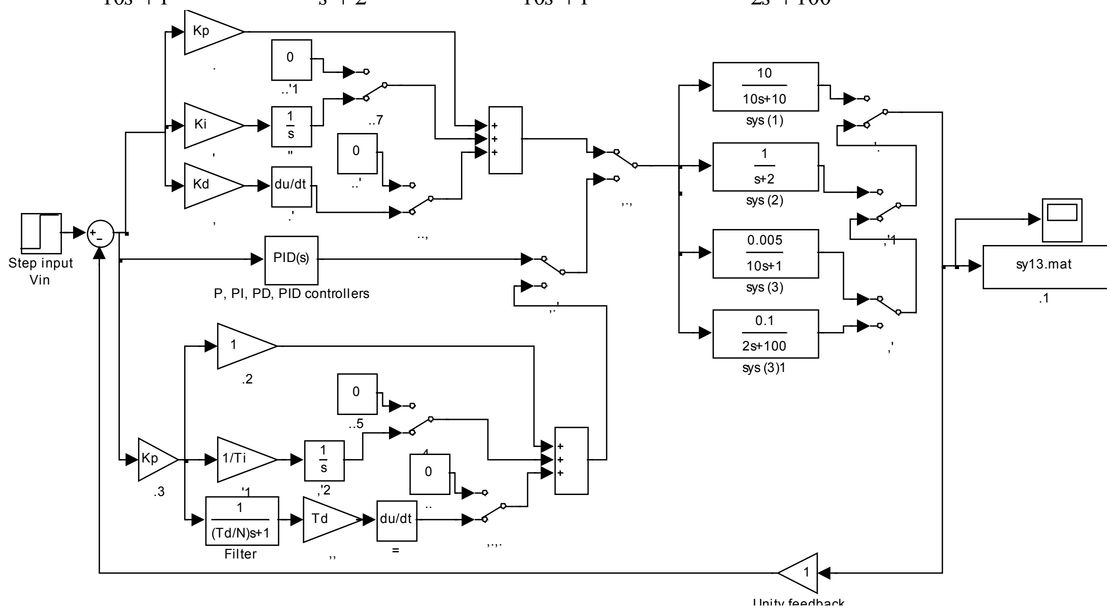


Figure 2 Simulink model for testing proposed design method

Table 1: P-controller design for first order system

P-Controller	α	System 1	System 2	System 3
		T=1	T=0.5	T=10
$K_P = \alpha RT$	α=1	10	5	100
	α=5	50	25	500
$R=V_{in}=10$	α=10	100	50	1000

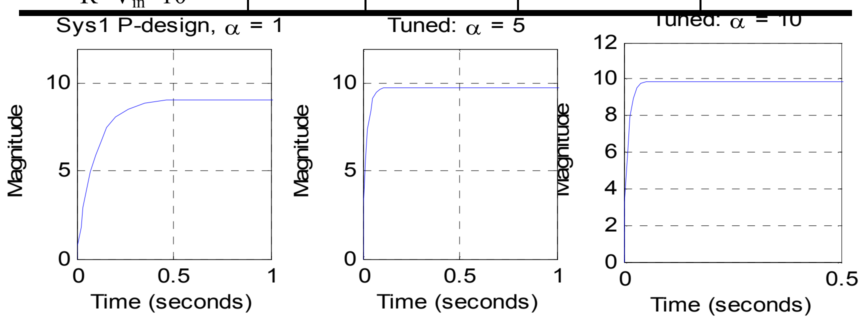


Figure 2(a) P-controller design for system 1

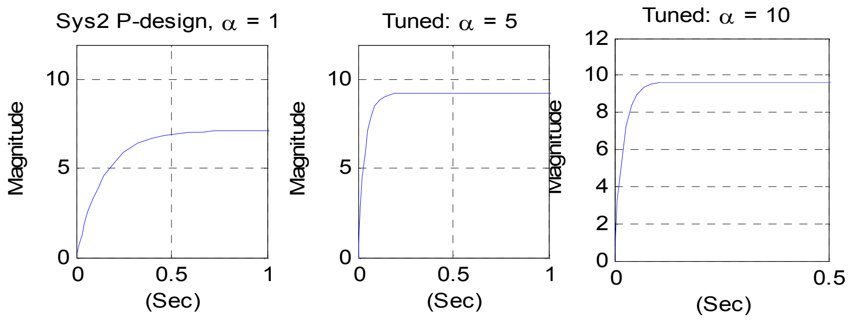


Figure 2(b) P-controller design for system 2

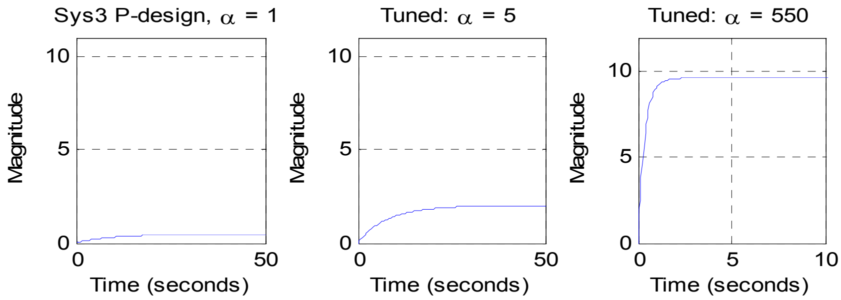


Figure 2(c) P-controller design for system 3 applying Eq.(17).

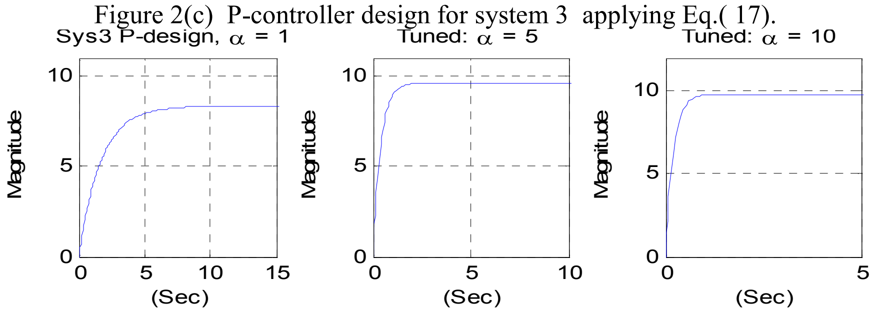


Figure 2(d) P-controller design for system 3 applying Eq.(18)

2.2 PI-controller design for first order systems

Based on plant's time constants, expression given by Eq.(20) are proposed, to design PI controller for first order systems, these expression relate selecting PI controller gains values based, only, on plant's time constant . To reduce overshoot and speed up response, soft tuning parameters α, is introduced.

To verify and test the proposed PI-controller design procedure, the same four first order systems are given in Eq.(19) and simulink model shown in Figure 1, are used, where manual switches are used to switch control system to PI controller. The result of designing PI-controller for achieving desired output of 10 with zero steady state error and minimum settling time, as well as, tuned values of tuning parameter α, are shown in Table 2, the response curves are shown in Figure 3, these response curves show, that a fast response, minimum overshoot and with zero steady state error are achieved. To speed up response, tuning parameter α can be increased softly, this is shown in Figure 3(b).

$$K_P = \alpha T$$
$$K_I = \frac{K_P}{T} = \frac{\alpha T}{T} = \alpha \qquad (20)$$
$$T_I = \frac{K_P}{K_I} = \frac{\alpha T}{\alpha} = T$$

Table 2: PI-controller design for first order system

	Parameters	System 1	System 2	System 3
PI-Controller	T	1	0.5	10
	α	1	1	1
	K_P	1	0.5	10
	K_I	1	1	10
	T_I	1	0.5	10
	N	1	1	1

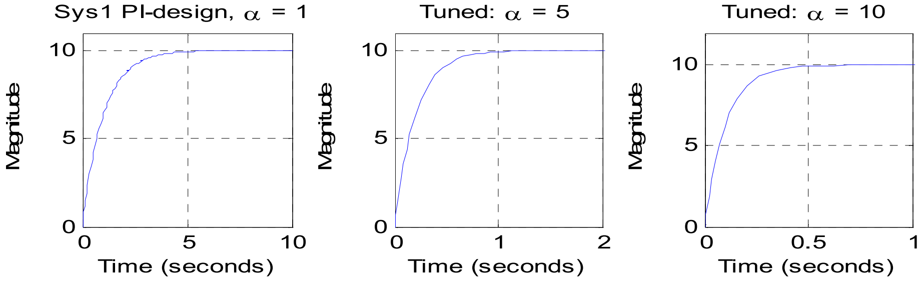


Figure 3(a) PI-controller design for system 1

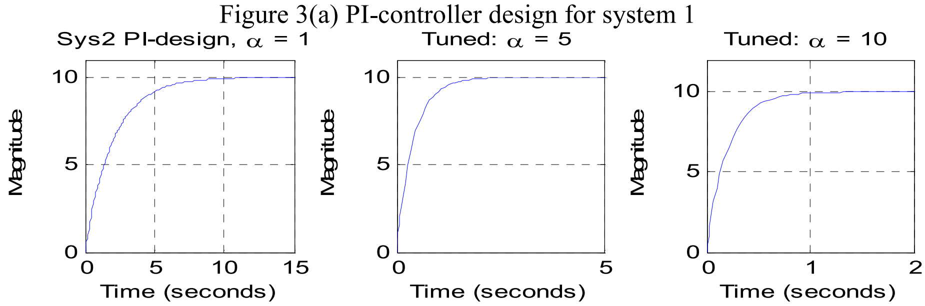


Figure 3(b) PI-controller design for system 2

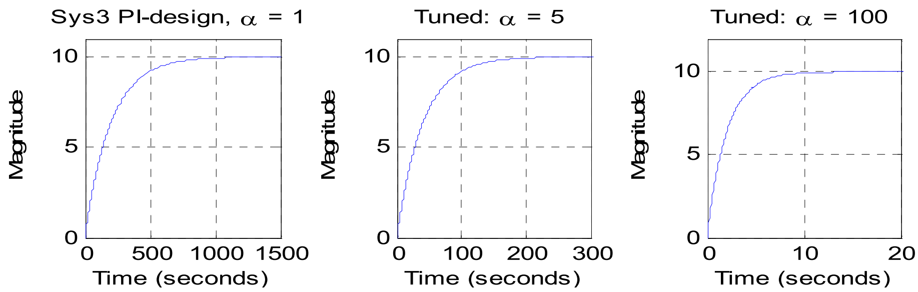


Figure 3(c) PI-controller design for system 3

2.3 PD-controller design for first order systems

Based on plants time constants, expressions given by Eq.(21) are proposed, to design PD controller for first order systems. For first order systems with small time constant *and/or* small DC gain, expression given by Eq.(22) are proposed, to reduce steady state error and speed up response, a soft tuning parameters α, is introduced .To verify and test the proposed PD-controller design procedure, the same first order systems given in Eq.(19) and simulink model shown in Figure 1 are used , manual switches are used to switch control system components to PD controller. The result of designing PD-controller for achieving desired output of 10 with minimum steady state error and minimum settling time, as well as, tuned values of tuning parameter alpha, are shown in Table 3, the response curves are shown in Figure 4, the response curves show, that a fast response with minimum steady state error are achieved. To speed up response and reduce steady state error tuning parameter α are increased, this is shown in Figure 4. Since system 3, is with small both DC gain and time constant, expressions given by Eq.(22) are applied, resulting in acceptable response compromise shown in Figure 4(b).

$$K_P = \alpha R T$$
$$K_D = \frac{T}{\alpha} \qquad (21)$$
$$T_D = K_D / K_P = 1/R\alpha$$

$$K_P = \alpha R^2 T$$
$$K_D = \frac{T}{\alpha} \qquad (22)$$
$$T_D = K_D / K_P = 1/R^2\alpha$$

Table 3: Applying PD-controller design for first order system

	Parameters	System 1	System 2	System 3
PD-Controller	T	1	0.5	10
	α	1	1	5
	K_P	10	5	5000
	K_D	1	.5	2
	T_D	0.1	0.1	4. e-004
	N	1	1	1

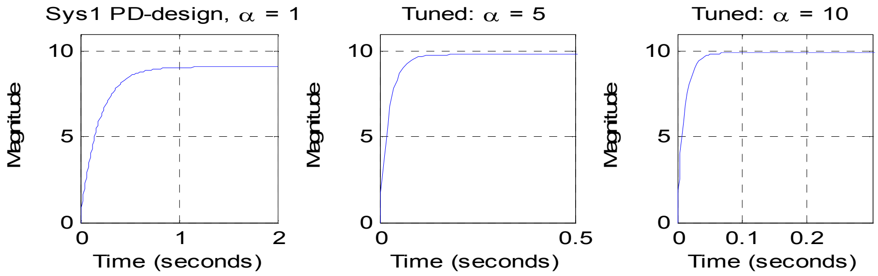


Figure 4(a) PD-controller design for system 1, increasing tuning parameter alpha reduces error and speeds response

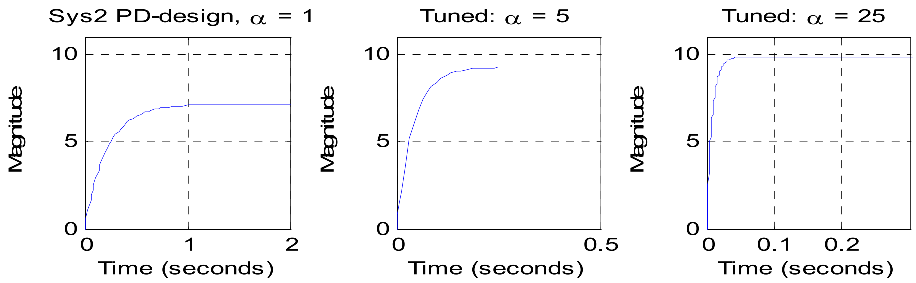


Figure 4(b) PD-controller design for system 2,

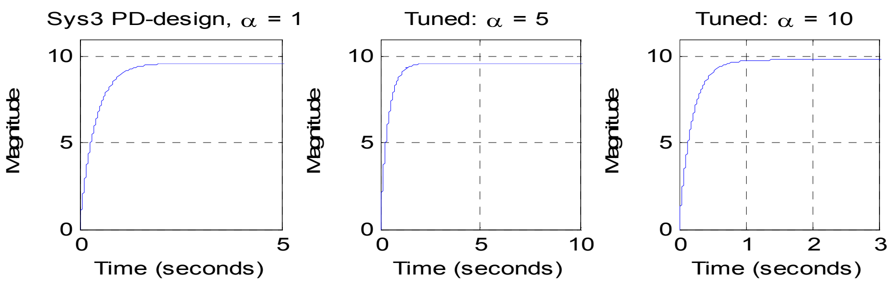


Figure 4(b) PD-controller design for system 3, applying expression given by Eq.(22)

2.4 PID-controller design for first order systems

Based on plants time constants, expressions given by Eq.(23) are proposed, to design PID controller for first order systems. Two tuning parameters are introduced α and ε, that can be tuned to speed up response, reduce overshoot and steady state error, based on methods applied for derivation (calculations, analysis and trial and error methods), a limits for these two parameters are proposed, these parameters are tuned softly, where: Tuning parameter α :is responsible for reducing overshoot , *(α* =0.1:3), reducing will increase damping and correspondingly, reduce overshoot, meanwhile, Tuning parameter ε : is responsible for speeding up response, *(ε* =0.1:2)

$$
\begin{aligned}
&K_P = T \\
&K_I = \alpha * T, \text{ where } \alpha = 0.1 \div 3 \\
&K_D = \varepsilon * T, \text{ where } \varepsilon = 0.1 \div 2 \\
&T_D = \frac{K_D}{K_P} = \frac{\varepsilon * T}{T} = \varepsilon \\
&T_I = \frac{K_P}{K_I} = \frac{T}{\alpha * T} = \frac{1}{\alpha}
\end{aligned}
\quad (23)
$$

To verify and test the proposed PID-controller design procedure, the same four first order systems are given in Eq.(19) and simulink model shown in Figure 1 are used , manual switches are used to switch control system to PID controller. The result of designing PID-controller for achieving desired output of 10 with minimum steady state error and minimum settling time, as well as, tuned values of tuning parameter alpha and epsilon, for system 1 are shown in Table 4, the response curves are shown in Figure 5, the response curves show, that a fast response with minimum both overshoot and steady state error at less time are achieved.

Table 4 PD-controller design for system 1, responses are shown in Figure 5(a)

		System 1	System 1	System 1
PD-Controller	T	1	1	1
	α	3	1	0.5
	ε	2	1	1
	K_P	1	1	1
	K_I	3	1	0.5
	K_D	2	1	1
	T_D	2	1	1
	T_I	0.333	1	2
	N	1	1	1

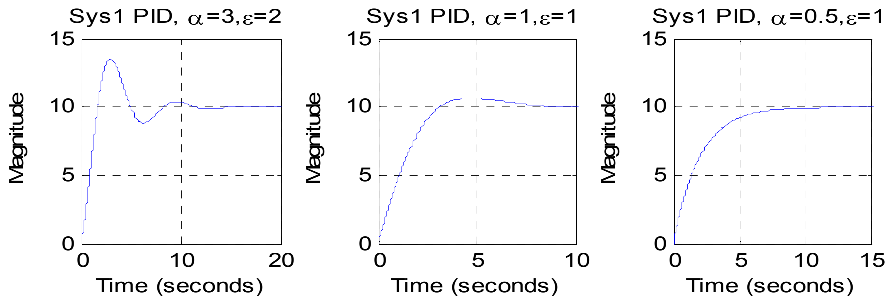


Figure 5(a) PID controller design for system 1, and the effect of tuning parameters ε and α

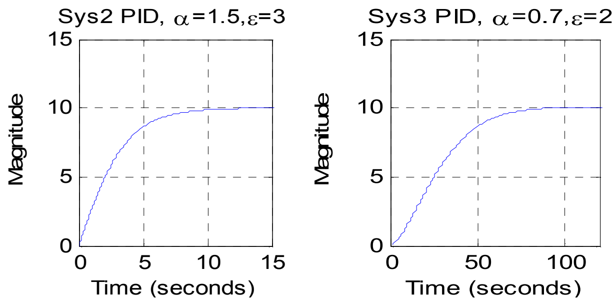


Figure 5(b) PID controller design for systems 2 and 3,

2.5 Summary : Controller design for First order systems

The proposed method and expressions for controllers terms selection and design for first order systems are summarized in Table 5

Table 5 P, PI ,PD, PID controllers terms for *first* order systems

Controller type	K_P	K_I	K_D	T_D	T_I	N
P-controller	RT	0	0	0	0	0
For sys. small DC gain and/or small T	αRT					
PD	RT	0	$K_D = T\ \alpha$	$K_D / K_P = 1/R\alpha$	0	$1 \div 22$
For sys. small DC gain and/or small T	$\alpha R^2 T$		$K_D = T\ \alpha$	$1/R^2\alpha$	0	0
PI	T	$\frac{K_P}{T} = \frac{\alpha T}{T}$ α	0	0	T	$1 \div 22$
PID	T	$*T$, $\alpha = 0.1 \div 3$	T, $\varepsilon = 0.1 \div 2$	$T_D = \frac{K_D}{K_P} = \frac{\varepsilon * T}{T} = \varepsilon$ $\varepsilon = 0.1 \div 2$	$T_I = \frac{K_P}{K_I} =$ $T_I = \frac{T}{\alpha * T} = \frac{1}{\alpha}$ $\alpha = 0.1 \div 3$	1-22

3. Controllers design for *second* order systems

The general standard form of second order system, in terms of damping ratio ζ and undamped natural frequency ω_n is given by Eq.(24), knowing that that performance of second order systems depends on damping ratio ζ and undamped natural frequency ω_n, where damping ratio determines how much the system oscillates as the response decays toward steady state and undamped natural frequency ω_n, determines how fast the system oscillates during any transient response (Farhan A. Salem, 2013), ω_n has a direct effect on the rise time, delay time, and settling time, therefore to speed up response and reduce (*remove*) overshot, based on this an important design compromise; acceptable stability, and medium fastness of response can be achieved by relating controller's gain and plant's parameters

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (24)$$

3.1 P-controller design for second order systems

Based on plant's damping ratio ζ and undamped natural frequency ω_n, expression given by Eq.(25) is proposed To design P-controller for second order systems. For systems with small time constant or DC gain expression given by Eq.(26) is proposed. Only one tuning parameter α is introduced, that can be tuned to speed up response, reduce overshoot, oscillation and steady state error and,

$$K_P = \frac{\alpha R\omega_n}{\xi}, \text{ where } \alpha \text{ tuned from } 1 \quad (25)$$

$$K_P = \frac{\alpha R^2\omega_n}{\xi}? \text{ where } \alpha \text{ tuned from } \quad (26)$$

Where R: is desired output value, α: can be increased for small R and decreased for large R. *increasing* parameter α will result in *increasing* overshoot, oscillation and reducing steady state error.
To test and verify the proposed P-controller design, four second order systems are given by Eq.(27) and simulink model shown in figure 6. Applying P-controller design for achieving desired output of 10, for three first

systems, will result in response curves shown in Figure 7, plants' parameters, tuned values of tuning parameter α and ε, gains and resulted performance specifications, are shown in Table 6, the response curves show, system will reach output with overshoot, oscillation and steady state error, increasing α will result in increasing overshoot, reducing steady state error.

$$G_{sys1}(s)=\frac{10}{10s^2+10s+2},\quad G_{sys2}(s)=\frac{1}{s^2+s+1}$$
$$G_{sys3}(s)=\frac{0.05}{2s^2+2s+1},\quad G_{sys4}(s)=\frac{7}{2s^2+5s+1} \qquad (27)$$

Table 6 plants' parameters, tuned values of tuning parameter α and ε, gains and performance specifications

	Parameters	System 1	System 1	System 2	System 3	System 4
P-Controller	**ζ**	0.5	0.5	0.3536	0.7071	2.5
	ω	1	1	1.4141	0.7071	1
	α	1	2	1	5	2
	R	10	10	10	10	10
	K_P	20	40	199	500	8
	T_S	7.8	6.8	7	8	2
	OS%	58%	73%	16.5%	16.5%	29%

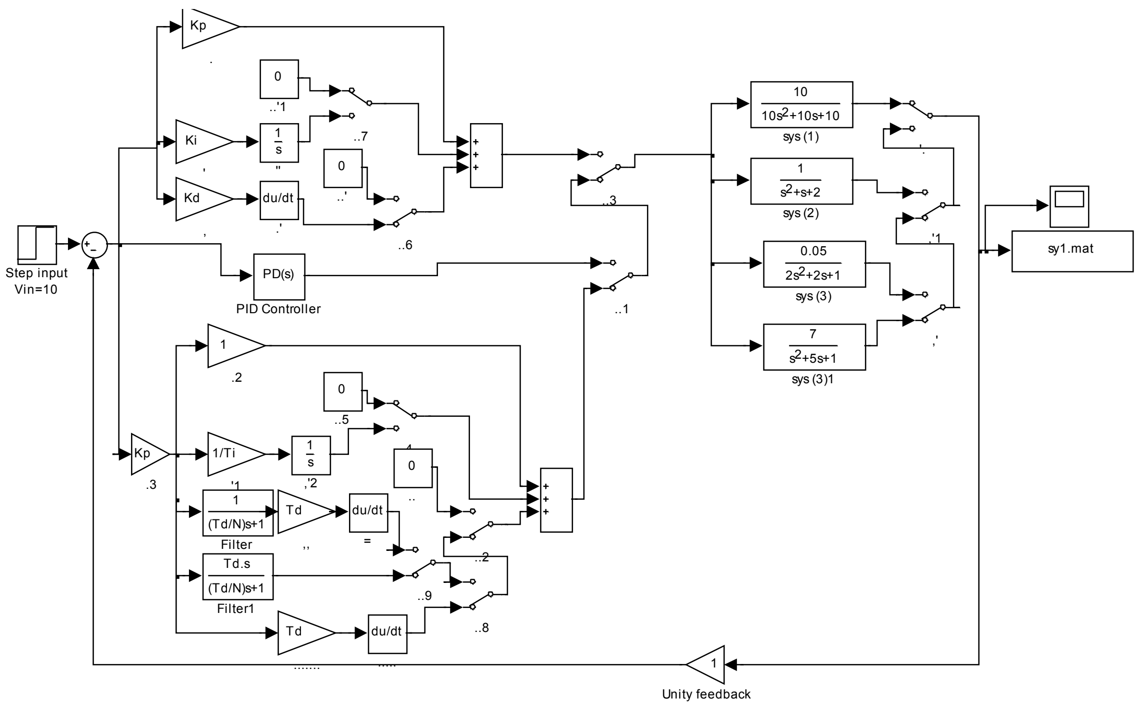


Figure 6 Simulink model for verifying design

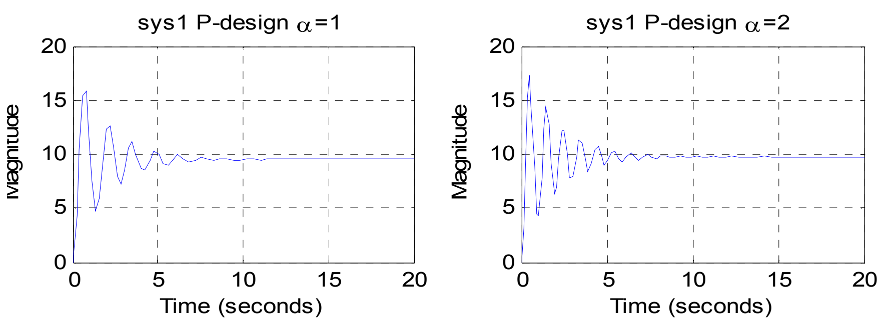


Figure 7(a) P-controller design for system 1, for α=1 , 2

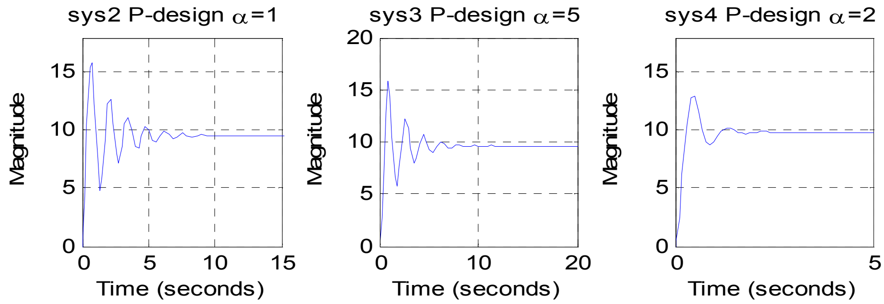


Figure 7(a) P-controller design for system 2,3,4

3.2 PD-controller design for second order systems

Based on plant's damping and undamped natural frequency, expression given by Eq.(28) are proposed to design PD-controller for second order systems. For systems with small time constant *and/or* small DC gain, expression given by Eq.(29) are proposed, in both cases, only *one* tuning parameter *α* is introduced, that can be softly tuned to speed up response, reduce overshoot and steady state error, where increasing parameter α will result in decreasing overshoot, oscillation and reducing steady state error.

Applying PD controller design for system 1 for achieving desired output of 10, and by tuning parameter α to have the values α =1,5,15, will result in response curves shown in Figure 8(a), the response curves show, increasing α will result in reducing steady state error and overshoot, while meeting acceptable response.

Applying PD-controller design for achieving desired output of 10, for system 2 , will result in response curves shown in Figure 8(b). Applying PD-controller design for achieving desired output of 10, for system 3 and applying expression given by Eq.(29) will result in response curves shown in Figure 8(c). These response curves show that a response with minimum overshoot and minimum error and with suitable settling time is achieved, also by soft tuning of only one parameter α, the response can be softened .

$$K_P = \frac{\alpha R}{\xi\omega_n} \qquad (28)$$
$$K_D = 2.9\alpha R\,\xi\omega_n$$
$$T_I = \frac{K_D}{K_P} = \frac{2.9\alpha R\,\xi\omega}{\frac{\alpha R}{\xi\omega_n}} = 2.9\xi^2\omega_n^2$$

$$K_P = \frac{\alpha R^2}{\xi\omega_n} \qquad (29)$$
$$K_D = \alpha R^2\xi\omega_n$$
$$T_I = \frac{K_D}{K_P} = \frac{\alpha R\,\xi\omega}{\frac{\alpha R}{\xi\omega_n}} = \xi^2\omega_n^2$$

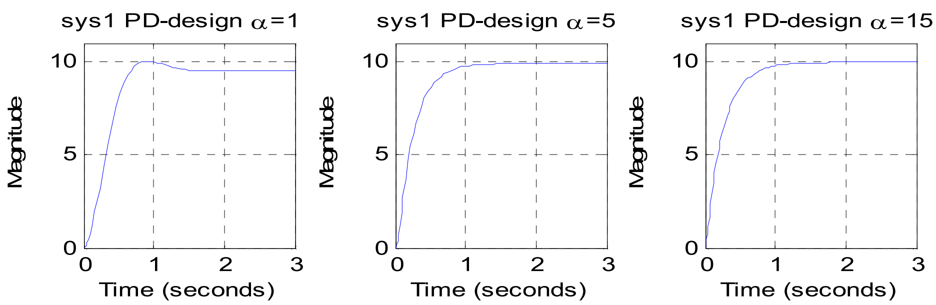


Figure 8 PD(a) controller design for system 1, for α=1,5,15

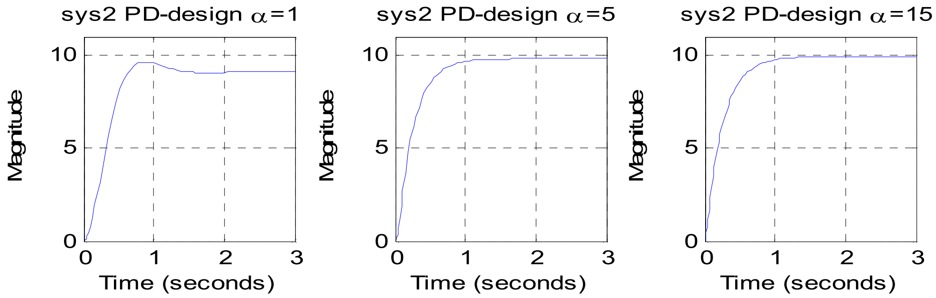


Figure 8(b) PD controller design for system 2, for α=1,5,15

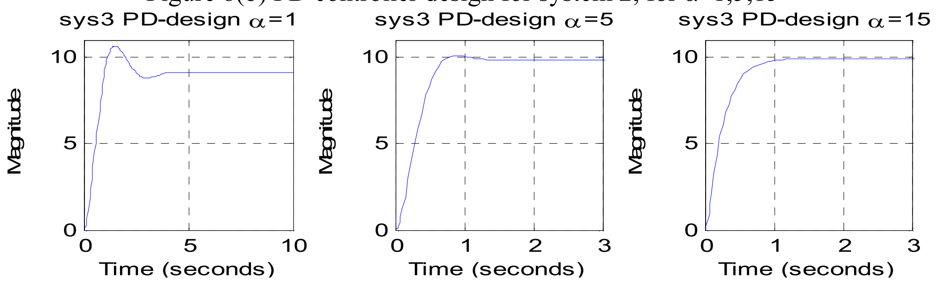


Figure 8(c) PD controller design for system 3, for α=1,5,15

3.3 PI-controller design for second order systems

To design PI-controller for second order systems, expression given by Eq.(30) are proposed, to speed up response, only one tuning parameter α is introduced.. Applying PI-design for second order systems given by Eq.(27), will result in response curves shown in Figure 9, the response curves show that an acceptable response with zero steady sate error and without overshoot are achieved.

$$\begin{aligned} K_I &= \frac{\xi+\omega_n}{10\xi\omega_n} \\ K_P &= \alpha K_I, \quad where \quad \alpha = .8 \div 1 \\ T_I &= \frac{K_P}{K_I} = \alpha \Rightarrow T_I = \frac{10\alpha\xi\omega_n K_I}{(\xi+\omega_n)} \end{aligned} \qquad (30)$$

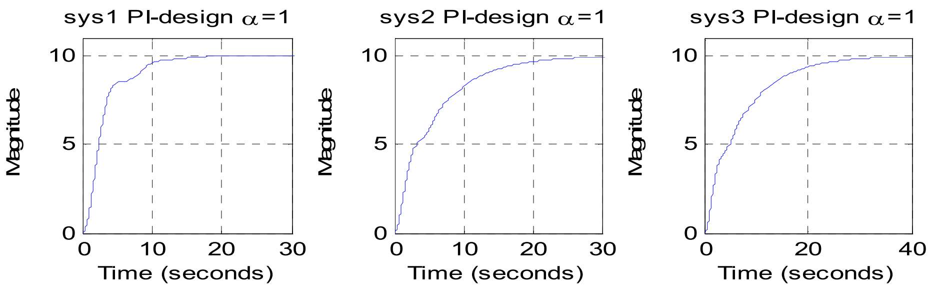


Figure 9 PI-controller design for three second order systems

3.4 PID-controller design for second order systems

The general standard form of second order system, can be expressed in terms of damping ratio ζ and undamped natural frequency ω_n as given by Eq.(31). Since PID transfer function is a second order system, it can also be expressed in terms of damping ratio and undamped natural frequency as given by Eq.(31), the PID gains ; K_P, K_I, K_D, can be found in terms of plant's damping ratio and undamped natural frequency, as given by Eqs.(32)(33). Assigning proportional gain, the value of unity, K_P=1, and equating Eqs.(31)(32), to find integral gain K_I, wil result in Eqs.(34) that is used to find numerical values of PID gains given by Eq.(35), based on plant damping ratio and undamped natural frequency, to soften the response in terms of reducing overshoot, steady state error and speed up response a tuning parameters ε and α with proposed rages are introduced.

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \Rightarrow G_{PID}(s) = \frac{K_D\left[s^2+2\xi\omega_n s+\omega_n^2\right]}{s} \qquad (31)$$

$$\omega_n^2 = \frac{K_I}{K_D} \Rightarrow K_D = \frac{K_I}{\omega_n^2} \qquad (32)$$

$$2\xi\omega_n = \frac{K_P}{K_D} \Rightarrow K_D = \frac{K_P}{2\xi\omega_n} \qquad (33)$$

$$\frac{K_P}{2\xi\omega_n} = \frac{K_I}{\omega_n^2} \Rightarrow \frac{1}{2\xi\omega_n} = \frac{K_I}{\omega_n^2} \Rightarrow K_I = \frac{\omega_n^2}{2\xi\omega_n} \Rightarrow K_I = \frac{\omega_n}{2\xi}$$

$$K_D = \frac{K_P}{2\xi\omega_n} \Rightarrow K_D = \frac{1}{2\xi\omega_n} \qquad (34)$$

$$K_P = 1$$

$$K_P = 1, \quad K_D = \frac{1}{2\xi\omega_n}, \quad K_I = \frac{\omega_n}{2\xi} \qquad (34)$$

Since PID transfer function, can be expressed in terms of derivative time and integral time to have the form given by Eq.(35) Where: $T_I = K_P / K_I$, the integral time and $T_D = K_D / K_P$, the derivative time

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) = K_P \frac{T_I T_D s^2 + T_I s + 1}{T_I s} \qquad (35)$$

$$K_I = K_P / T_I \quad , \quad K_D = K_P T_D$$

Based on this, the derived formulae for calculating PID controller gains in terms of derivate time T_D and integral time T_I , to be as given by Eq.(36), the divisor *N* is chosen in the range 2 to 20.

$$T_I = \frac{K_P}{K_I} = \frac{K_P}{\omega_n / 2\xi} = \frac{K_P 2\xi}{\omega_n} = \frac{2\xi}{\omega_n}$$

$$(36)$$

$$T_D = \frac{K_D}{K_P} = \frac{1/2\xi\omega_n}{K_P} = \frac{1}{2\xi\omega_n} = K_D$$

For soft tuning of proposed PID controller design, two tuning parameters are introduced α and ε , Where: Increasing ε will increase overshoot, and vise versa, while increasing α will speed response, as a result, tuning these parameters will result in reducing overshoot, reducing steady state error and speeding up response. The proposed expressions for PID controller design are listed in Table 7.

Testing proposed PID controller design method for second order systems given by Eq.(27), will result in PID terms, given in Table **8 ,** and response curves shown in Figure 10. These response curves show an acceptable response without overshoot, steady state error and in acceptable settling time are achieved.

Further testing PID controller design procedure for second order systems given by Eq.(37), will result in PID terms, tuned parameters values given in Table **9 ,** and response curves shown in Figure 11 .

$$sys1 = \frac{1}{s^2 + s + 1}, \quad sys2 = \frac{1}{s^2 + 6s + 5}, \quad sys3 = \frac{1}{10s^2 + 10s + 40} \qquad (37)$$

Table 7 Proposed expressions for PID gains calculation

Plant		PID parameters					
		K_P	K_I	K_D	T_D	T_I	N
ζ	ω_n	1	$\frac{\omega_n}{2\xi}$	$\frac{1}{2\xi\omega_n}$	$\frac{1}{2\xi\omega_n}$	$\frac{2\xi}{\omega_n}$	2 ÷ 20
Tuning limits		1	$\varepsilon\frac{\omega_n}{2\xi}, \varepsilon = .1 \div 10$	$\alpha\frac{1}{2\xi\omega_n}, \alpha = .58 \div 1.5$	$\frac{\alpha}{2\xi\omega_n}$	$\frac{2\xi}{\varepsilon\omega_n}$	

Table 8 Testing results of second order systems

System	Plant parameter				PID parameters		
parameter	**ζ**	**ω_n**	**ε**	**α**	**K_P**	**K_I**	**K_D**
Sys(1)	0.5	1	0.62	1.1	1	0.62	1.1
Sys(2)	0.3536	1.4142	0.68	1.5	1	1.3598	1.4998
Sys(3)	0.7071	0.7071	8	1	1	4	1

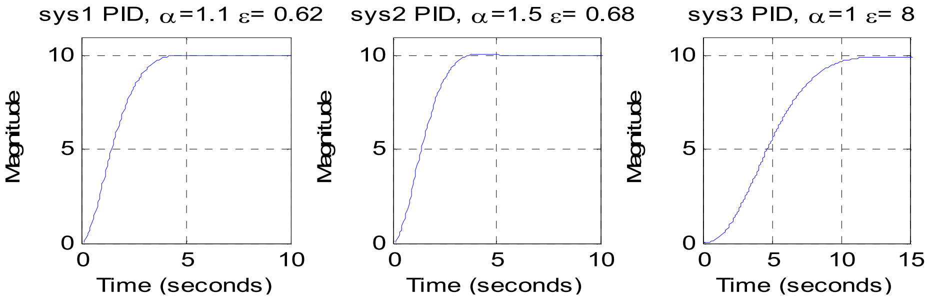
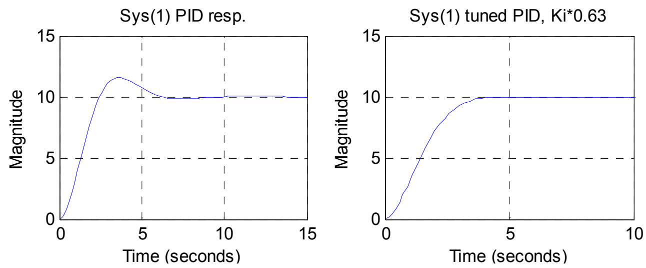


Figure 10applying PID control method for three second order systems

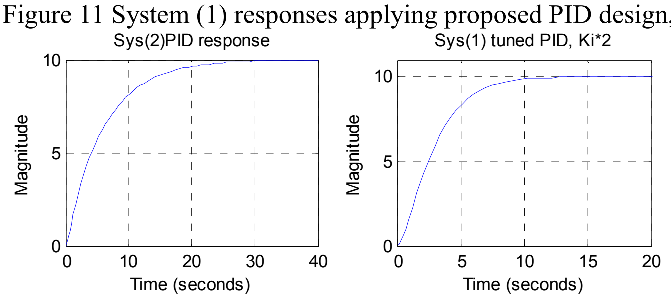
Table 9 Testing results of second order systems

System	**Plant parameter**		**PID parameters**		
parameter	ζ	ω_n	K_P	K_I	K_D
Sys(1)	0.5	1	1	1	1
Sys(1), tuned *only* K_I value			1	0.6300	1
Sys(2)	1.3416	2.2361	1	0.8333	0.1667
Sys(2), tuned *only* K_I value			1	1.6667	0.1667
Sys(3)	0.2500	2	1	4	1
Sys(2), tuned *only* K_I value			1	8	1



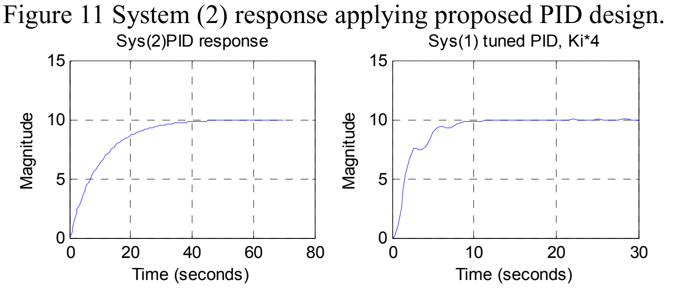
(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 11 System (1) responses applying proposed PID design,



(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 11 System (2) response applying proposed PID design.



(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 11 System (3) response applying proposed PID design

3.5 Summary: Controller design for second order systems

The proposed method for second order systems and expressions for controllers terms selection and design are summarized in Table 10

Table 10 P, PI , PD, PID controllers terms for *second* order systems

Controller type	K_P	K_I	K_D	T_D	T_I	N
P-controller	$\frac{\alpha R\,\omega_n}{\xi}$	0	0	0	0	0
For systems with small DC gain *and/or* small T	$\frac{\alpha R^2\omega_n}{\xi}$					
PD	$\frac{\alpha R}{\xi\omega_n}$	0	$2.9{*}\alpha R\,\xi\omega_n$	$2.9\xi^2\omega_n^2$	0	1-22
Systems with small DC gain *and/or* small T	$\frac{\alpha R^2}{\xi\omega_n}$	0	$\alpha R^2\xi\omega_n$	$\xi^2\omega_n^2$	0	1-22
PI	αK_I $\alpha = 0.8 \div 2$	$\frac{\xi+\omega_n}{10\xi\omega_n}$	0	0	$T_I = \frac{K_P}{K_I} = \alpha \Rightarrow$ $T_I = \frac{10\alpha\xi\omega_n K_I}{(\xi+\omega_n)}$	1-22
PID	1	$\varepsilon\frac{\omega_n}{2\xi}$,牋 $= .1 \div 2$	$\alpha\frac{1}{2\xi\omega_n}$?牋 $\alpha = .58 \div 1.5$	$\frac{\alpha}{2\xi\omega_n}$	$\frac{2\xi}{\varepsilon\omega_n}$	1-22

4. PID Controller for first order plus delay time (FOPDT) process

A large number of industrial plants can approximately be modeled by a first order plus time delay (FOPTD) (Katsuhiko Ogata, 2010)(Saeed Tavakoli et al, 2003). FOPDT models are a combination of a first-order process model with dead-time ,it transfer function is given by Eq.(38) and it response curve is shown in Figure 12, this s-shape curve with no overshoot *is called reaction curve, it* is characterized by two constants ; the delay time *L*, and time constant *T*, these two constants can be determined by drawing a tangent line at the inflection point of the s-shaped curve, and finding the intersection of the tangent line with time axis and steady state level *K*, (see Figure 12), then the transfer function of these-shaped curve can be approximated by first order system with transport lag and given by Eq.(38)[]:

$$\frac{C(s)}{R(s)} = \frac{Ke^{-Ls}}{Ts+1} \qquad (38)$$

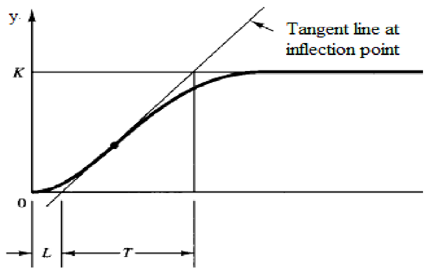


Figure 12 s-shaped curve with terminology (Farhan A. Salem, 2013)

Based on plant's delay time L, time constant T, and steady state level *K*, (see Eq. (12)). The formulae listed in

Table 11, are proposed to calculate PID gains in terms L, T, K for (FOPDT) process and tuning limits for K_D and K_I. Based on Eqs.(35)(36), the derived formulae for calculating PID controller gains in terms of derivate time T_D and integral time T_I , to be as given by in Table 11, the divisor N is chosen in the range 2 to 20.

Table 11 Proposed formulae for PID gains calculation for first (FOPDT) system, and softening ranges

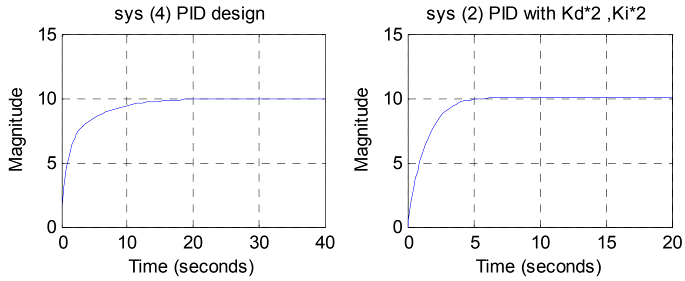
Plant	PID parameters					
	K_P	K_I	K_D	T_I	T_D	N
T,K,L	T	$L*T$	$\frac{L}{T}$	$1/L$	$\frac{L}{T^2}$	$2 \div 20$
Tuning limits	T	$\varepsilon*L*T$, $\varepsilon = 0.1 \div 3$	$\alpha\frac{L}{T}$ $\alpha = 0.1 \div 3$	$\frac{1}{\varepsilon*L}$	$\alpha\frac{L}{T^2}$	

Applying proposed PID design method for first-order process with dead-time given by Eq.(39), will result in the calculated PID gains values listed in Table 12, the response curve is shown in figure 13

$$\frac{C(s)}{R(s)} = \frac{e^{-0.3s}}{s+1} \tag{39}$$

Table 12 design for FOPDT

System	Plant's parameters	PID parameters							
FOPDT	**Parameters**	**α**	**ε**	K_P	K_I	K_D	T_D	T_I	**N**
	L=0.3,T=1,K=1	2	2	1	0.60	0.6	6.6666	0.6	2



(a) Applying calculated PID gains (a) after tuning K_D and K_I

Figure 13 FOPDT response applying proposed PID design,

5. Comparing and testing proposed design with existing design methods

Case (1): Considering a third order plant with transfer function given by Eq.(40), to verify proposed design, it will be compared with Ziegler-Nicols design method

$$G(s) = \frac{1}{(s+1)^3} \tag{40}$$

Since this is third order system, it can be approximated as second order system with two repeated pole $P=1$, Correspondingly ζ=1, ω =1, designing P, PI, PD, and PID controller applying Ziegler-Nicols design method and proposed design will result in gains values listed in table 13, and the response curves are shown in Figure 14 , these response curves show that the proposed method is more simpler than Ziegler-Nicols, as well as a more smooth response with minimum overshoot and acceptable settling time is achieved

Table 13 controllers' gains values applying both Ziegler-Nicols and proposed design methods

	Parameters		Pro.	PD		PI		PID		
Proposed method	ζ	ω	K_P	K_P	K_D	K_P	K_I	K_P	K_I	K_D
	1	1	5	20	58	0.1	0.1	1	0.5	0.5
Ziegler-Nicols method	K_{crit}	P_{crit}	K_P	K_P	T_D	K_P	T_I	K_P	T_I	T_D
	8	3.62	4.8	-	-	3.6	3	4.8	1.8	0.45

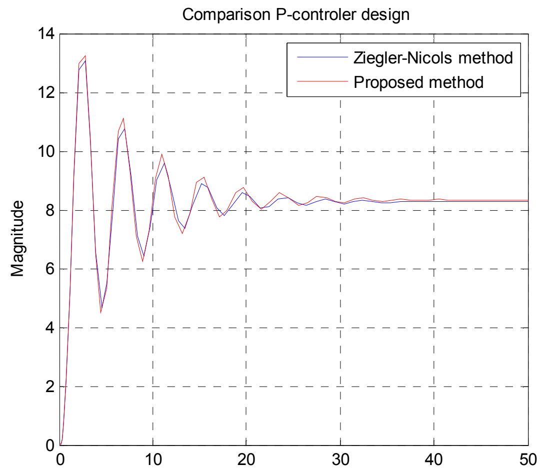


Figure 14(a) P-controller design applying Ziegler-Nicols and proposed design methods

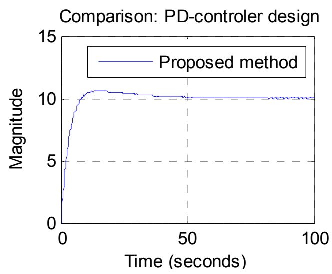


Figure 14(b) PD-controller design applying proposed design methods

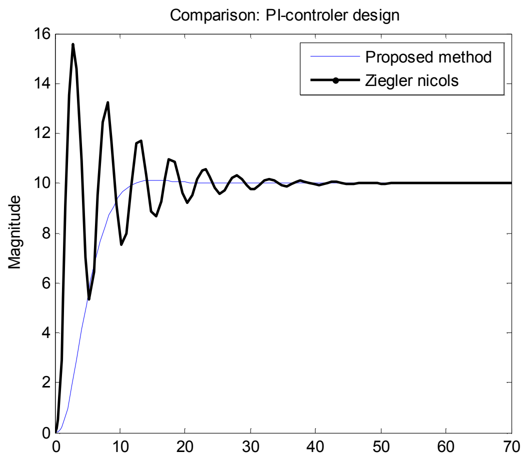


Figure 14(c) PI-controller design applying Ziegler-Nicols and proposed design methods

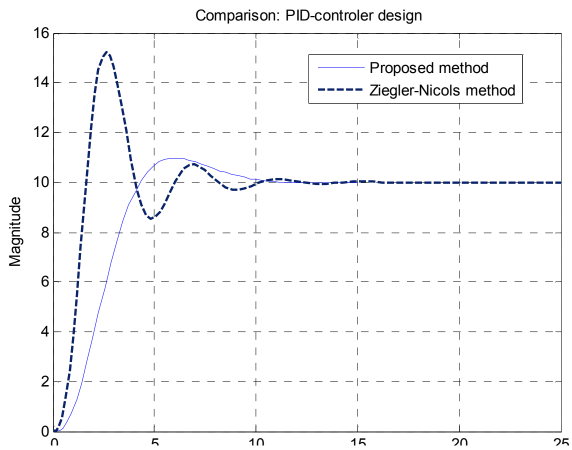


Figure 14(c) PID-controller design applying Ziegler-Nicols and proposed design methods

Case (2): Testing proposed PID design method for fourth order plant transfer function given by Eq.(41), applying three different PID controller design methods, particularly, Ziegler Nichols frequency response, Ziegler-Nichols step response, and Chein-Hrones-Reswick design methods, will result in PID gains shown in Table 14(Robert A. Paz, 2001), , as shown in this table different values of PID gain are obtained and correspondingly different system's responses (see figure 15), when subjected to step input of 10. Comparing shown response curves, show that the Chein-Hrones-Reswick design is, with less overshoot and oscillation (than Ziegler-Nicols), all three method allmostly, result in the same settling time, Applying the proposed method, based on plant's dominant poles approximation, result in smooth response curve without overshoot, and zero steady state error, shown in figure 14.

$$G(s) = \frac{10000}{s^4 + 126s^3 + 2725s^2 + 12600^s + 10000} \quad (41)$$

Table 14

Design Method	K_P	K_I	K_D
Ziegler Nichols Frequency Response	14.496	45.300	1.1597
Ziegler-Nichols Step Response	11.1524	34.3786	0.9045
Chein-Hrones-Reswick	5.5762	5.0794	0.4522
Proposed method	1	0.8632	0.1231
	ζ=3.0677	ω_n= 2.6481	

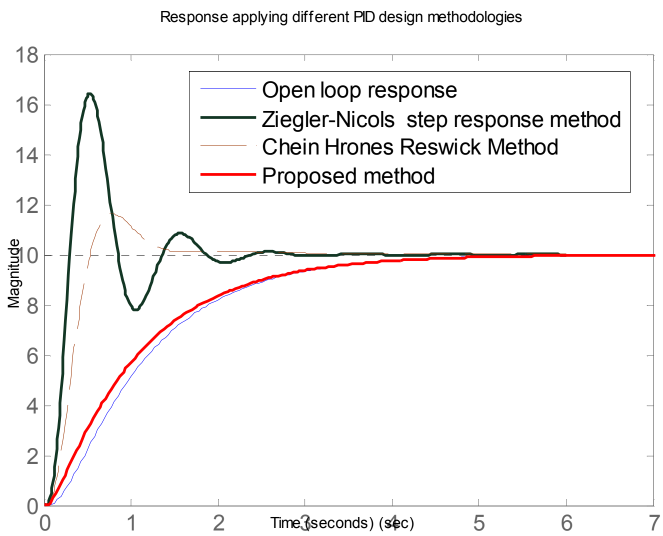


Figure 15 System step responses obtained applying different design methodologies

Conclusion

A new simple and efficient model-based time domain P, PI ,PD , and PID controllers design method for achieving an important design compromise; acceptable stability, and medium fastness of response is proposed, the proposed method is based on selecting controllers' gains based on plant's parameters, the proposed controllers design method was test for first, second and first order system with time delay, and using MATLAB/simulink software.

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